高维概率

High-Dimensional Probability

四、应用：随机图模型

@滕佳烨

- **切尔诺夫不等式**

对称伯努利分布->霍弗丁不等式->Gaussian-like bound $\exp(-ct^2)$

中心极限定理：对称伯努利<->正态

伯努利分布->切尔诺夫不等式->Possian-like bound

当p比较小，用泊松近似二项分布更加紧！

- 切尔诺夫不等式

Theorem 2.3.1 (Chernoff's inequality). *Let X_i be independent Bernoulli random variables with parameters p_i. Consider their sum $S_N = \sum_{i=1}^N X_i$ and denote its mean by $\mu = \mathbb{E} S_N$. Then, for any $t > \mu$, we have*

$$\mathbb{P}\left\{S_N \geq t\right\} \leq e^{-\mu}\left(\frac{e\mu}{t}\right)^t.$$

- **本章知识点总结（独立随机变量和的Concentration）**

次高斯分布->次高斯模->霍弗丁不等式→$\exp(-ct^2)$

次指数分布->次指数模->伯恩斯坦不等式→$\exp(-ct)$

非渐进、指数收敛

- **应用：随机图模型（切尔诺夫不等式）**

模型背景（无向图）
G(n, p):n个结点，每个节点间连接的概率为p
度(degree)：平均连接数，$d=(n-1)p$

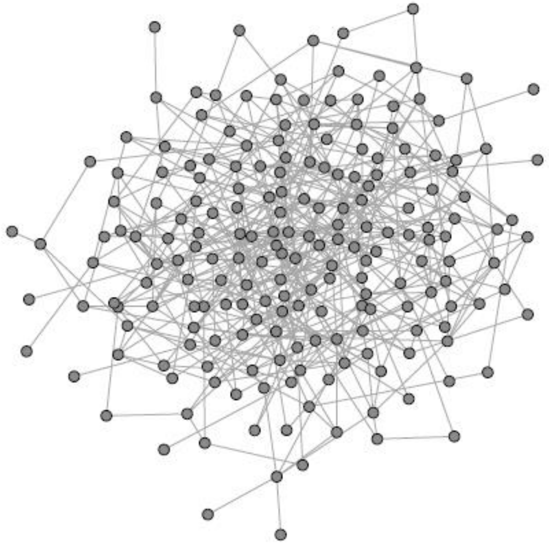

Figure 2.2 A random graph from Erdös-Rényi model $G(n,p)$ with $n=200$ and $p=1/40$.

- **应用：随机图模型（切尔诺夫不等式）**

结论一：在稠密图(dense graph)是规则图(regular)

Proposition 2.4.1 (Dense graphs are almost regular). *There is an absolute constant C such that the following holds. Consider a random graph $G \sim G(n,p)$ with expected degree satisfying $d \geq C \log n$. Then, with high probability (for example, 0.9), the following occurs: all vertices of G have degrees between $0.9d$ and $1.1d$.*

Hint:首先使用切尔诺夫不等式，任意一个结点大概率都有d个左右的连接，再使用union bound，将其拓展到所有节点中。

注意到最后一句中的**all**

- **应用：随机图模型（切尔诺夫不等式）**

结论二：稀疏图

Exercise 2.4.2 (Bounding the degrees of sparse graphs). ☕ Consider a random graph $G \sim G(n, p)$ with expected degrees $d = O(\log n)$. Show that with high probability (say, 0.9), all vertices of G have degrees $O(\log n)$.

Exercise 2.4.3 (Bounding the degrees of very sparse graphs). ☕☕ Consider a random graph $G \sim G(n, p)$ with expected degrees $d = O(1)$. Show that with high probability (say, 0.9), all vertices of G have degrees

$$O\Big(\frac{\log n}{\log \log n}\Big).$$

- **应用：随机图模型（切尔诺夫不等式）**

结论二：稀疏图(upper bound)

Exercise 2.4.2 (Bounding the degrees of sparse graphs). ☕ Consider a random graph $G \sim G(n,p)$ with expected degrees $d = O(\log n)$. Show that with high probability (say, 0.9), all vertices of G have degrees $O(\log n)$.

Exercise 2.4.3 (Bounding the degrees of very sparse graphs). ☕☕ Consider a random graph $G \sim G(n,p)$ with expected degrees $d = O(1)$. Show that with high probability (say, 0.9), all vertices of G have degrees

$$O\Big(\frac{\log n}{\log\log n}\Big).$$

- 应用：随机图模型（切尔诺夫不等式）

结论三：稀疏图(lower bound)

Exercise 2.4.4 (Sparse graphs are not almost regular). ☕☕☕ Consider a random graph $G \sim G(n,p)$ with expected degrees $d = o(\log n)$. Show that with high probability, (say, 0.9), G has a vertex with degree[2] $10d$.

Exercise 2.4.5 (Very sparse graphs are far from being regular). ☕☕ Consider a random graph $G \sim G(n,p)$ with expected degrees $d = O(1)$. Show that with high probability, (say, 0.9), G has a vertex with degree

$$\Omega\Big(\frac{\log n}{\log\log n}\Big).$$

谢谢！

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